

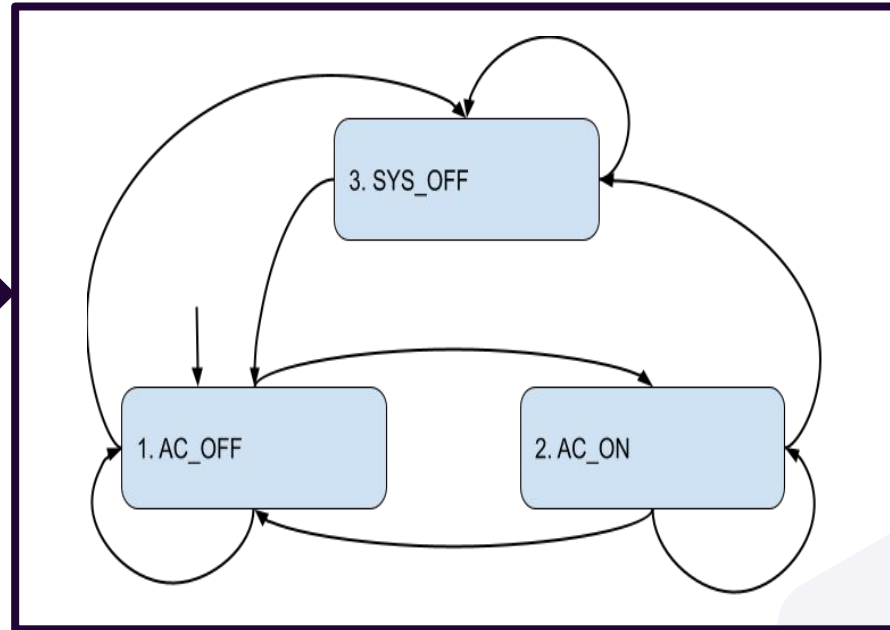
# 30: Hybrid systems





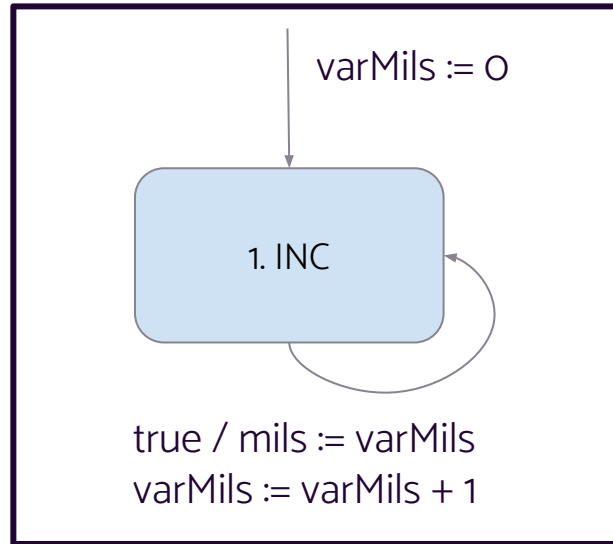
# AC model from prelab

on/off button  
currTemp  
desTemp  
mils





# Modeling mils



mils



*How do we model:*

- *Current temp*
- *Desired temp*
- *Button push*

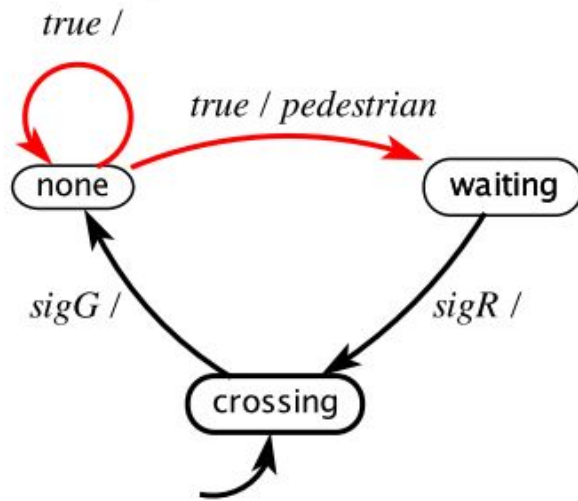
# Nondeterminism

Guards out of a state are not mutually exclusive

- Reason about the “possible set of states” a system can be in
- Useful for modeling environment

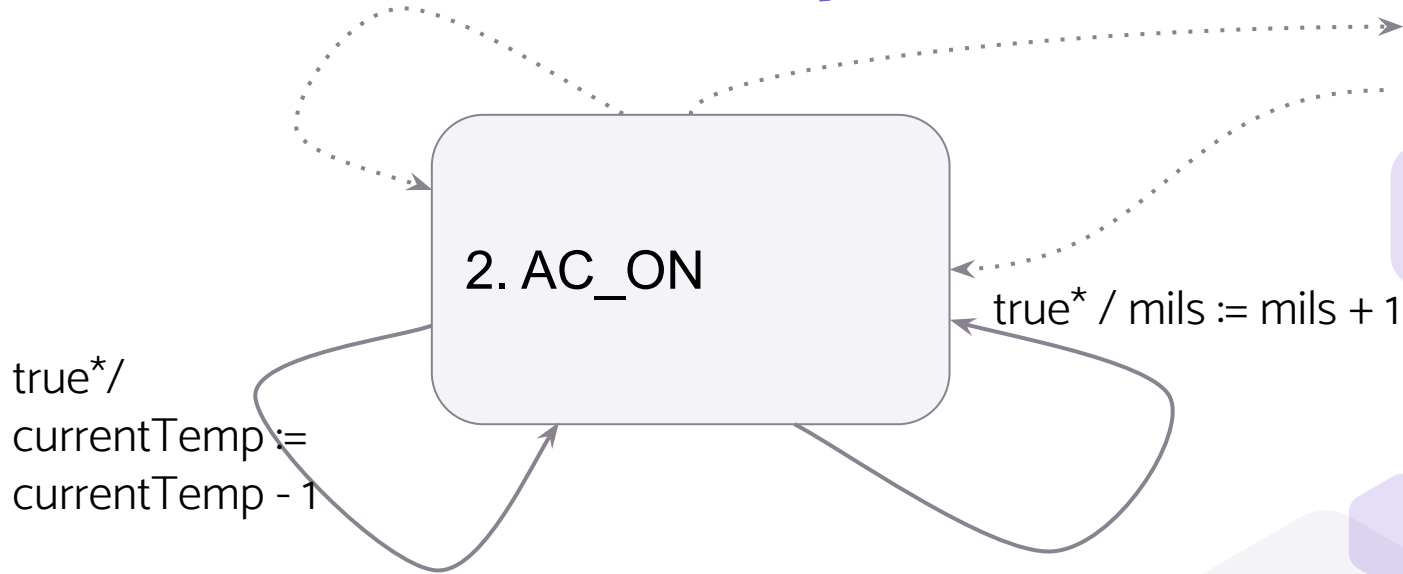
**inputs:**  $sigR, sigG, sigY$  : pure

**outputs:**  $pedestrian$  : pure



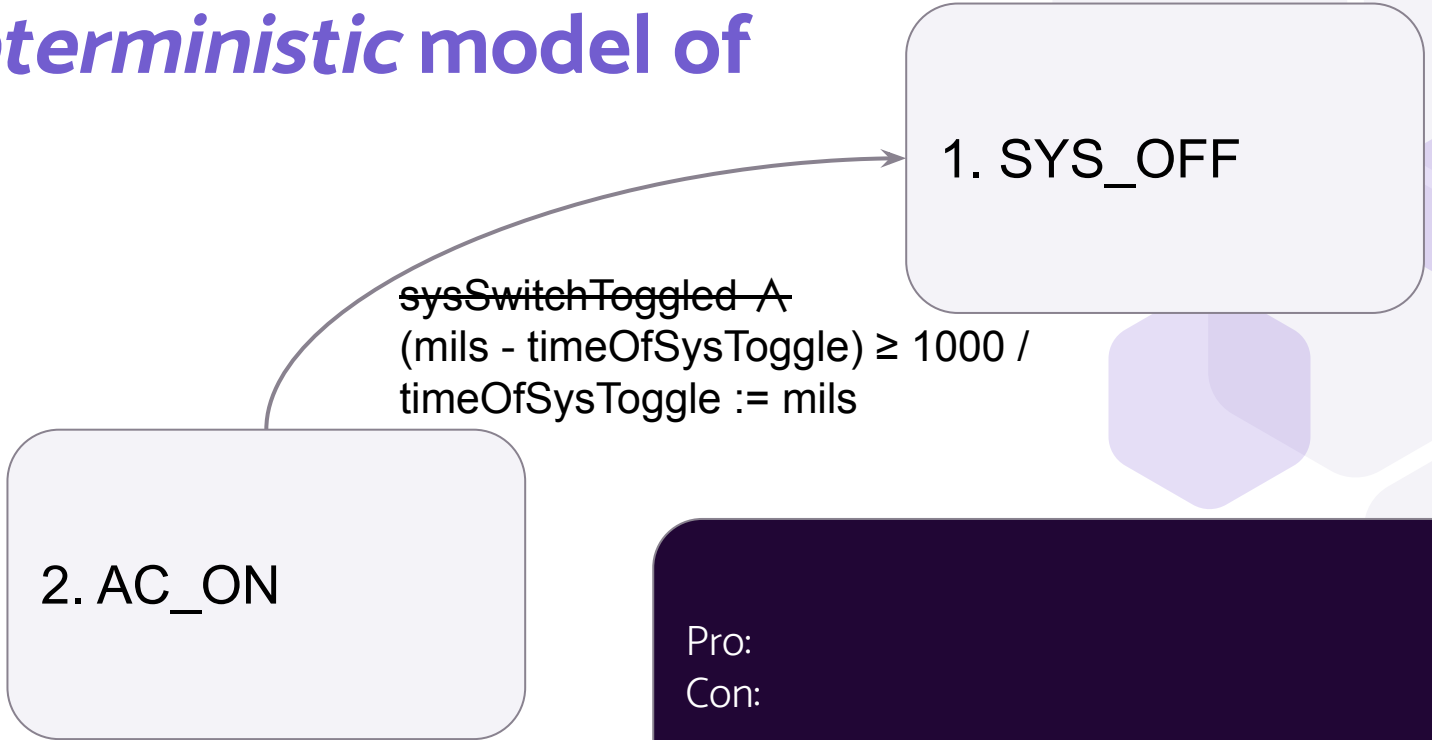
Lee/Seshia fig. 3.11

# AC model composed with very simplified, *non-deterministic* model of current temp and time

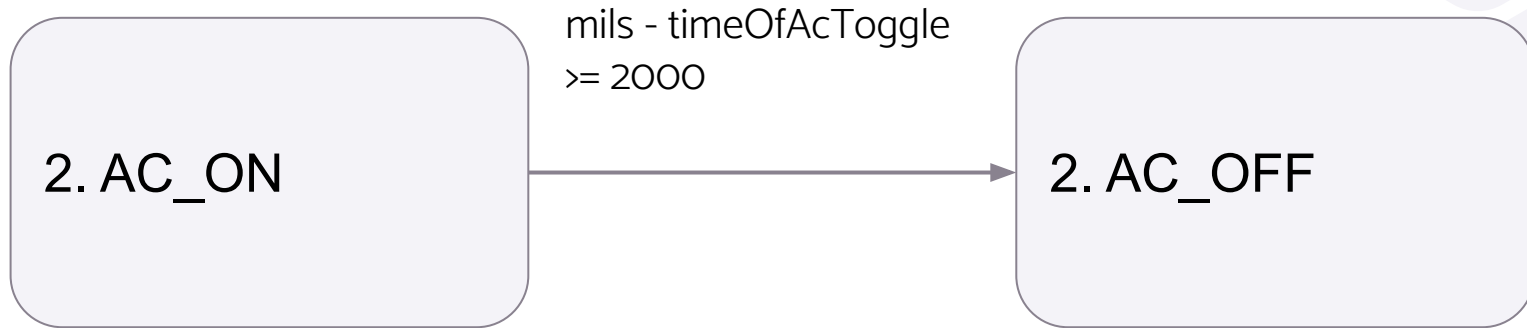


*\*we would actually have to make sure that these transitions aren't taken if we need to leave state 2, e.g.  $\neg(\text{sysSwitchToggled} \wedge (\text{mils} - \text{timeOfSysToggle}) \geq 1000)$*

# AC model composed with *non-deterministic* model of button



# Composing with model of desired temp: option 1

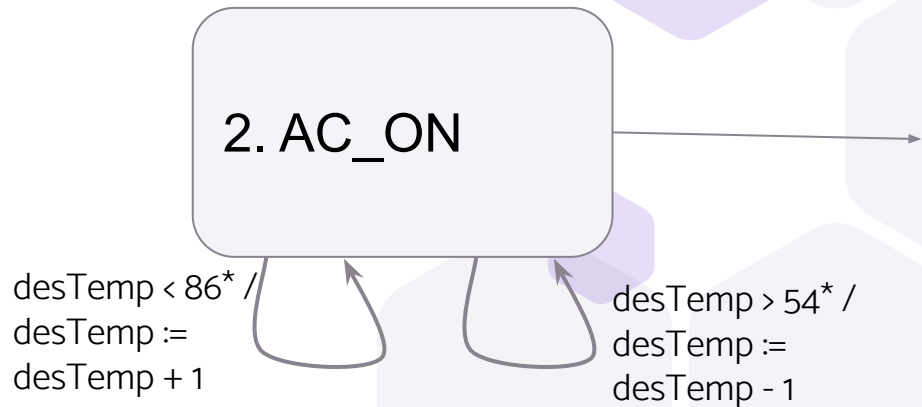
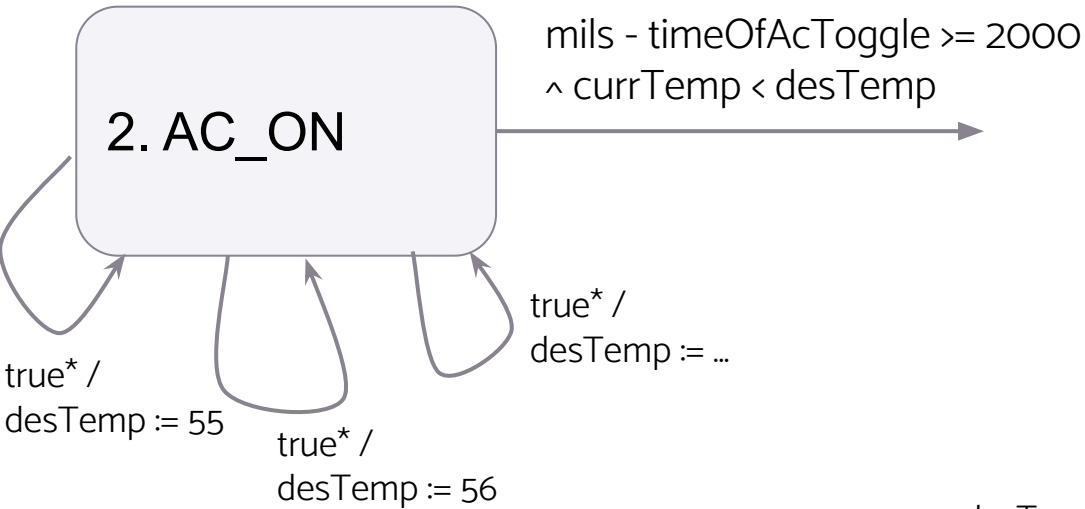


We lose ability to prove things like:  
(`desTemp > curTemp` → eventually `AC_ON`)





# Composing with model of desired temp: options 2 and 3





*What are we missing out on  
when we tell time by using  
“mils” as an input?*



# ODEs

Sometimes it is more desirable to describe a variable in terms of how it changes rather than its explicit form

Useful for: modeling, reasoning

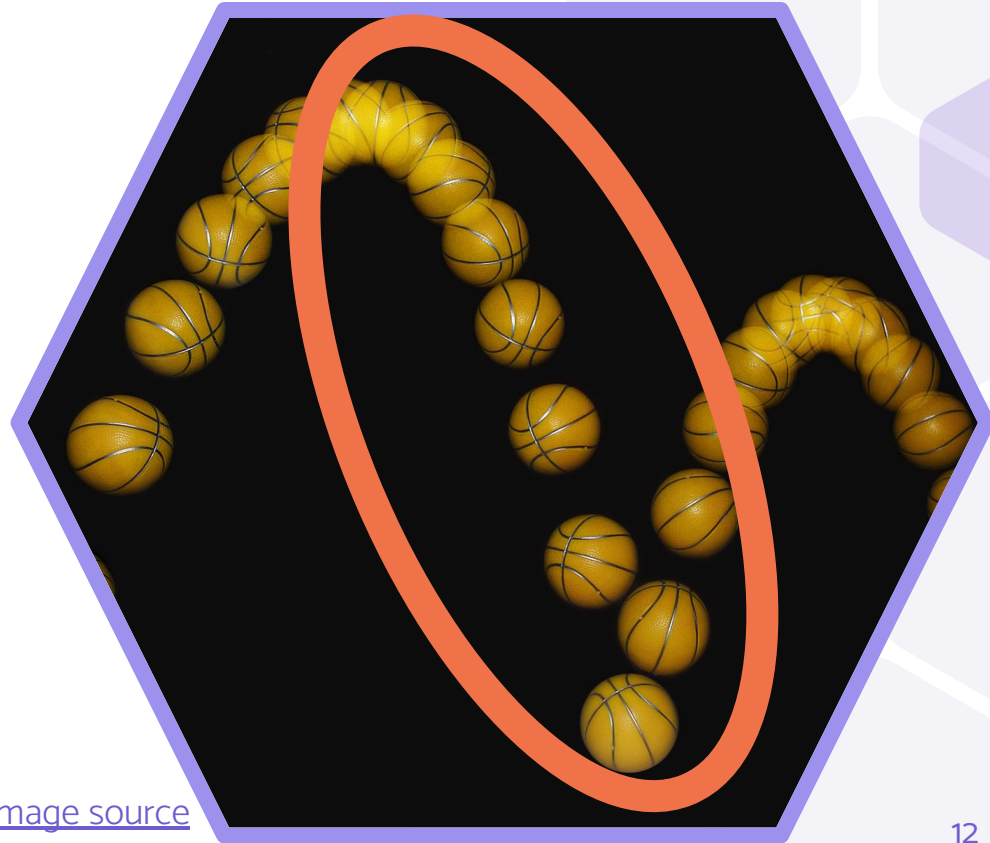
Define a function in terms of its derivative and possibly initial conditions

Ordinary Differential Equation, or ODE

Solving general ODEs is beyond the scope of the class, but we will discuss some patterns here



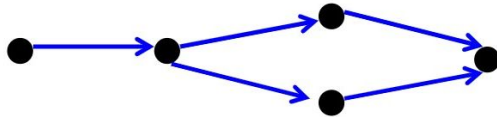
# Example: falling ball



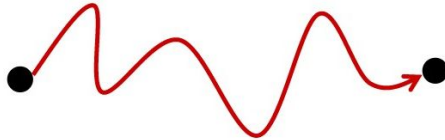
[Image source](#)

# Hybrid systems

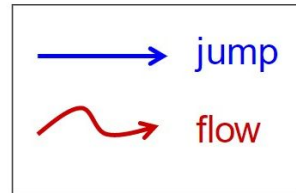
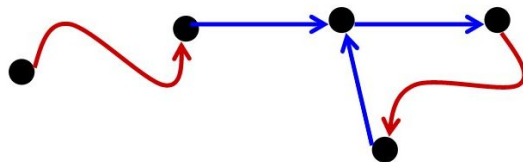
Discrete System (FSM)



Continuous System



Hybrid System





# Timed automata

Distinction between discrete and continuous variables

Continuous behavior defined in “states”

Now called “modes”

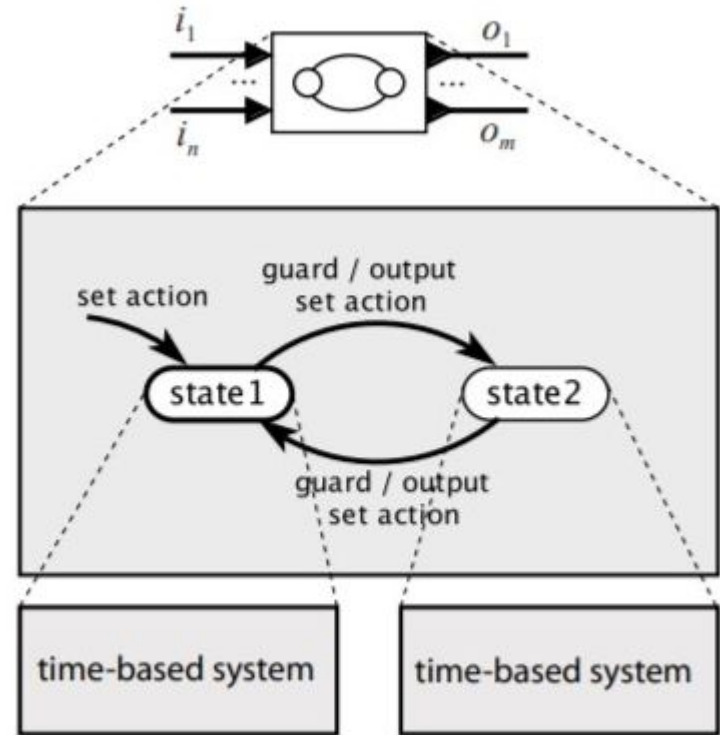


Figure 4.4: Notation for hybrid systems.

# Example: bouncing ball

Board discussion



[Image source](#)



# Discussion of homework problems